conclusions of this study are as follows: 1) at buoyancy numbers below about 5, the fuel volume fraction depends strongly on density ratio, but is independent of vehicle acceleration; 2) at buoyancy numbers larger than about 30, the fuel volume fraction is nearly independent of density ratio, and decreases with the $-\frac{2}{3}$ power of buoyancy number; 3) for a typical engine flow with a fuel-to-propellant density ratio of 10 and buoyancy number of 350, the fuel volume fraction may be reduced considerably due to buoyancy effect; 4) because of the relatively low Reynolds number of the analysis, the precise values of fuel volume fraction must be determined experimentally. Such experiments should be conducted at buoyancy numbers ranging from 50 to 500.

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Analytical Design of Optimal Nutation Dampers

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Nomenclature

H = gyro momentum
I_i = moment of inertia about principal axis i ($i = 1, 2, 3$)
J = gyro moment of inertia about output axis
l = distance between axis 3 and ball in equilibrium position
m = mass of the ball
q = damping coefficient
r = constant defined by Eq. (1)
R = radius of curvature of tube
s = Laplace variable
α = relative angle between damper and body
Γ = spring constant
ζ = viscous damping constant
ρ = coupling factor
ω = $(I_3 - I_1)\,\omega_3/I_1$, satellite nutation frequency
ω_i = inertial angular velocity of satellite along axis i (axis 3 = spin axis)

Superscripts

$(\ )'$ = differentiation with respect to s
$(\ )^*$ = optimal values

Introduction

PASSIVE nutation damping of spinning satellites leads naturally to the practical problem of determining optimal parameters values. Zajac[1] and Sarychev[2] use the "maximum damping rate" criterion for the case of gravity-gradient stabilized satellites. The purpose of this Note is to show how this criterion can be applied to spinning satellites. An analytical approach is used for the practical examples of a single-degree-of-freedom gyro and a ball-in-tube damper.

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Optimality Criterion

Let us assume the system under consideration has pervasive damping, so that the Hamiltonian can be used as a Liapunov function whose time derivative is $\Sigma\, Q_i \dot{q}_i$ (see Ref. 4).

For our problem this derivative is simply $-q\dot{\alpha}^2$, so that the speed of nutation decay depends only on q and $\dot{\alpha}$. For a passive damper, $\dot{\alpha}$ is maximum when the damper and satellite are tuned. In order to obtain the optimal damping coefficient q^*, we shall examine the linearized system.

Let us assume that ω_3 is constant and that the damper has only one degree of freedom. Then the linear set of equations describing the system motion is

$$\dot{\omega}_1 = -\omega\omega_2 + \rho(\alpha_{11}\omega_1 + a_{12}\omega_2 + a_{13}\dot{\alpha} + a_{14}\alpha) \quad (1a)$$

$$\dot{\omega}_2 = \omega\omega_1 + \rho(a_{21}\omega_1 + a_{22}\omega_2 + a_{23}\dot{\alpha} + a_{24}\alpha) \quad (1b)$$

$$\ddot{\alpha} = -q\dot{\alpha} - r\dot{\alpha} + a_{31}\omega_1 + a_{32}\omega_2 \quad (1c)$$

where $\rho \ll 1$ is a coupling factor between the satellite and the damper and a_{ij} are constant coefficients. The characteristic equation of this system can be written as

$$f(s) = (s^2 + \omega^2)(s^2 + qs + r) + \rho(as^4 + qbs^3 + cs^2 + qds + e) \quad (2)$$

The solutions of the linear system can be written in the form

$$A_1 \exp(-\sigma_1 t)\cos(\omega_1 t + \varphi_1) + A_2 \exp(-\sigma_2 t)\cos(\omega_2 t + \varphi_2)$$

If we assume that $q \ll \omega$ and that the system is tuned, that is $r \simeq \omega^2$, the roots of $f(s)$ are very close to $+i\omega$, so that A_1 is approximately equal to A_2 and we must have $\sigma_1 = \sigma_2$ for optimal damping. If we remove the assumption of small q, it can be easily shown that statistically A_1 and A_2 have the same importance, hence we must still have $\sigma_1 = \sigma_2 = -(q/4)$. But increasing q while keeping the tuning condition will lead to an increase in the size of damper thus posing practical limitations.

Assuming that the roots of $f(s)$ are close to $\pm i\omega$ an exact root can be found using a Taylor expansion

$$f(i\omega) + f'(i\omega)ds + \tfrac{1}{2}f''(i\omega)ds^2 + 0(ds^3) = 0 \quad (3)$$

As the system is tuned (i.e., $r^* = \omega^2$), $f'(i\omega)ds$ and $f''(i\omega)ds^2$ have the same order of magnitude; thus neglecting ds^2 in Eq. (3) would lead to a large error.

A simple computation shows that Eq. (3) can be approximated by the equation with real coefficients

$$-4\omega^2 ds^2 - 2q\omega^2 ds + \rho(a\omega^4 - c\omega^2 + e) = 0$$

so that $ds = -(q/4)$ leads to

$$q^* = 2[\rho(-a\omega^4 + c\omega^2 - e)]^{1/2}/\omega \quad (4)$$

with

$$r^* = \omega^2 \quad (5)$$

It should be noted that q is a real positive number because an application of the Routh-Hurwitz criterion to $f(s)$ gives approximately $-a\omega^4 + c\omega^2 - e > 0$ as a stability condition.

It should also be noted that q can be obtained just by writing

$$f(s) \simeq (s^2 + sq^*/2 + \omega^2)^2 = 0$$

which could lead to an extension to higher order systems.

Practical Examples

The ball in tube damper of Fig. 1 can be described by the equation (see Ref. 3)

$$\dot{\omega}_1 + \omega\omega_2 + \tfrac{1}{2}\rho(\ddot{\alpha} + \alpha\omega_3{}^2)/g = 0$$
$$\dot{\omega}_2 - \omega\omega_1 = 0$$
$$\ddot{\alpha} + q\dot{\alpha} + g\omega_3{}^2\alpha + 2g(\dot{\omega}_1 + \omega_2\,\omega_3) = 0$$

where $q = 5\,\zeta/7m$, $g = 5l/7R$, $\rho = 10m\,l^2/7I_1$.

With this set of equations, Eqs. (2, 4, and 5) reduce to

$$f(s) = (s^2 + \omega^2)(s^2 + qs + g\omega_3{}^2) - \rho(s^2 + \omega_3{}^2)(s^2 + \omega\omega_3)$$
$$q^* = 2|\omega - \omega_3|\,[\rho(\omega + \omega_3)/\omega]^{1/2}$$
$$r^* = g\omega_3{}^2 = \omega^2$$

For a given satellite, ω, ω_3 are known, so that l, m, R, ζ, must be found. The optimal damping and tuning conditions gives $l/R =$ const, $\zeta/lm^{3/2} =$ const. As the time constant T of the system is $4/q^*$ we can add a new relation, namely, $Tm^{1/2}\,l =$ const.

In the case of a single-degree-of-freedom gyroscope system, the axis of the CMG makes an angle α with axis 3 and the output axis is along axis 1 with spring and damper about it. The system linearized equations are

$$I\,(\dot{\omega}_1 + \omega\omega_2) - \Gamma\alpha - \zeta\dot{\alpha} = 0$$
$$I\,(\dot{\omega}_2 - \omega\omega_2) - (\omega_1 + \dot{\alpha})H + J\omega_3\dot{\alpha} = 0$$
$$J\,(\ddot{\alpha} + \omega_1) + \zeta\dot{\alpha} + (\Gamma + H\omega_3)\alpha + H\omega_2 = 0$$

assume $\rho = J/I$, $q = (1 + \rho)\,\zeta/J$, $K = (1 + \rho)\,\Gamma/J$, $h = H/J$; then neglecting terms in ρ^2, it comes

$$q^* = 2|\omega - h|\,[\rho(\omega + \omega_3)/\omega]^{1/2},\; r^* = K + h\omega_3 = \omega^2$$

It appears that one way of decreasing the time of nutation decay is to have a high spinning gyro, but with H negative. We can only fear instability if ω or ω_3 moves. The parameters to be found are H, J, Γ, ζ and we can get three relations as in the previous example.

Simulations and Results

For both systems, we have used a numerical procedure in order to check the analytical results. This numerical procedure is a search for the minimum time constant. On Fig. 2, the broken curve shows a step by step research of extremum. Curves of Figs. 2 and 3 give examples of optimum, and except for large q, computed points by analytical and numerical procedures are identical. Numerical values are taken from the French Meteosat satellite project and from technical literature.

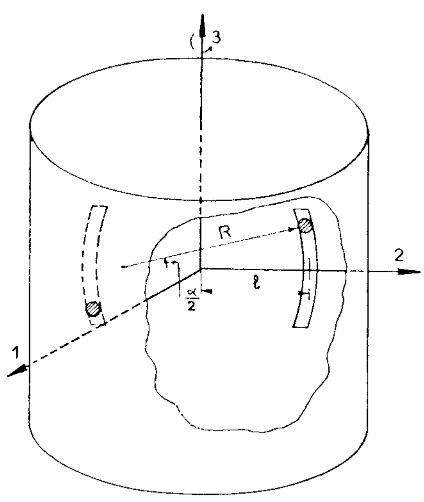

Fig. 1 Ball in tube damper geometry.

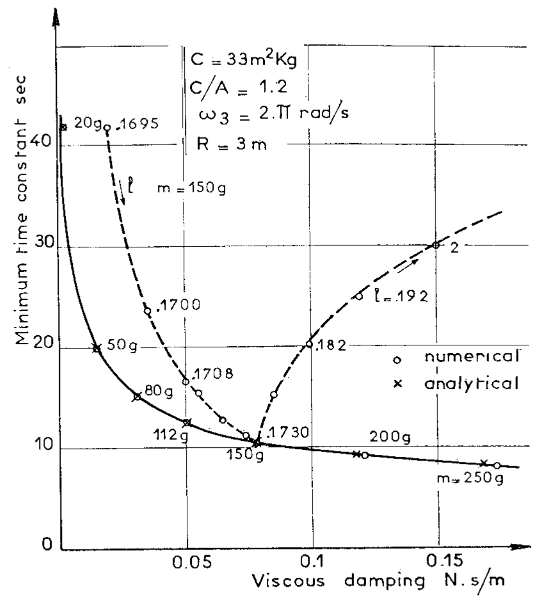

Fig. 2 Ball in tube damper time constant for maximum damping rate.

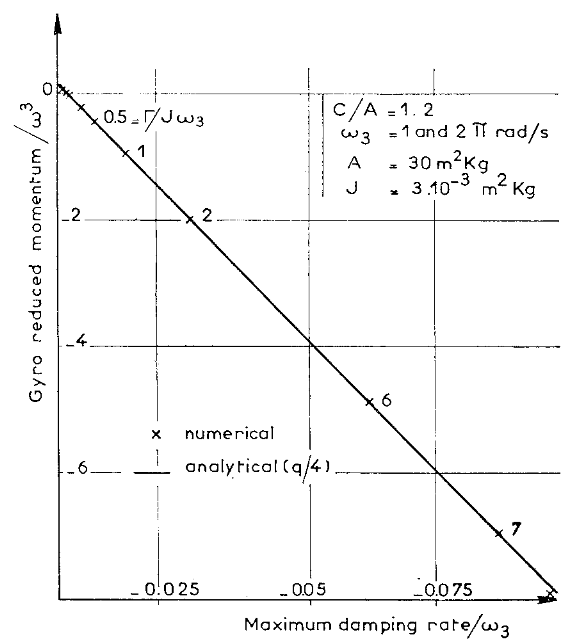

Fig. 3 Semipassive C.M.G.: variation of maximum damping rate.

From the examples considered, the method developed here appears as a powerful tool either for the practical design of dampers or to compare the efficiency of different dampers: one needs only to get Eq. (1) to deduce Eqs. (3) and (4).

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